# Design of Problem-Posing Exercise for Efficient Calculation

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**Abstract:** This paper describes a computer-supported posing exercise of numerical expressions that can be solved by using efficient calculation methods. This exercise is designed to promote students in an elementary school to change their procedural interpretation to relational interpretation of numerical expressions because the efficient calculation methods request learners to interpret the numerical expressions relationally.

**Keywords:** Procedural Interpretation, Relational Interpretation, Numerical Expression, Efficient Calculation Methods, Efficient Calculation Problem, Problem-Posing

## 1. Introduction

This paper describes a computer-supported posing exercise of numerical expressions that can be solved by using efficient calculation methods. Efficient calculation methods that make calculation easier by transferring a numerical expression, for example, changing calculation order (Cooper et al., 1996, Heirdsfield, 2004) are one of learning targets in an elementary school. Since it is an exercise that a learner has to pay attention to the relations in a numerical expression, it is expected to be effective for relaxing the difficulty in introduction phase of mathematics in a junior high school which reported in several researches (Booth, 1998). Several investigations suggest that posing a problem is a promising exercise to promote learners to think about the relations included in the problem (Silver, 1994, English, 1988). Moreover, in problem-posing, immediate feedback for the posed problems is effective to promote the learning (Nakano et al., 1999, Hirashima et al., 2007, Kojima et al., 2013). Based on these considerations, we have designed and developed a computer-supported posing exercise environment of efficient calculation problems. The environment has ability to automatically diagnose posed numerical expressions and generate feedback for based on the diagnosis results. So, in the exercise, a learner is able to receive immediate feedback for his/her posed problems.

## 2. Procedural Interpretation and Relational Interpretation of Numerical Expression

Because of the major change of the interpretation of the expressions at the introductory phase of mathematics in a junior high school, many students feel difficult to learn it (Booth, 1998). In order to relax this gap and smoothly connect to mathematics learning, it is promising to let students experience relational interpretation of numerical expression in arithmetic learning. Efficient calculation methods that are one of teaching targets in an elementary school request a learner to interpret a numerical expression as relations. Problem-posing of efficient calculation problems is a learning method which cannot be solved by executing procedural operation, and it is a relational operation of the numerical expressions at the same time. In order to effectively conduct the problem-posing, diagnosis of correct answers and appropriate feedback based on diagnosis are important. Unlike solving problems, multiple correct answers, an interactive learning system with diagnosis function of posed problems is indispensable.

# 3. Problem-Posing Exercise System

### 3.1 System Interface

The interface of the system developed in this research is shown in Fig. 1. The system consists of four exercises, (Exercise 1) problem solving exercise that only requests an answer of calculation, (Exercise 2) problem solving exercise that requests to write efficient calculation process, (Exercise 3) problem-posing exercise that requests to pose a problem that can be calculated with the same efficient calculation method used in exercise 2, and (Exercise 4) problem-posing exercise that requests to pose a problem that cannot be applied the efficient calculation method used in the previous exercises. In this exercise system, for one numerical expression, the four exercises are assigned. In Figure 2, the four exercises deal with "27 + 9" as the numeric expression, and targeting efficient calculation methods can be shown as "(27 + 3) + (9 - 3).



Figure 1. Interfaces of Exercises.

# 3.2 Details of Exercises

Fig. 2 shows examples of the four exercises constituting the problem-posing learning of efficient calculation methods. Exercise 1 is a calculation exercise that requests a learner only a correct answer and does not matter how the answer was derived. Exercise 2 is an efficient calculation exercise requests a learner to calculate following an example. Exercise 3 is a solution-based problem-posing exercise. The efficient calculation method of the learning target is presented as an example, and students pose a problem that can be solved by exemplified efficient calculation method. If the problem itself, the calculation process, and the final answer are all correct, the posed problem is regarded as a correct answer. Exercise 4 is an efficient calculation problem-posing (negative example). Unlike the efficient calculation method is not applicable, students only pose a problem and do not make the calculation process and final answer.



Preparation for understanding the effect of Efficient Calculation

Exercise	3: Problem-posing	exercise

Example	Le <u>arner's answer example</u>
27 + 8	27 + 6
=(27+3)+(8-3)	=(27+3)+(6-3)
= 30+5	= 30+3
= 35	= 33

Make a problem that can solve the same way as example and calculate it



Example 27 + 8	Learner's answer example 27 + 9
=(27+3)+(8-3)	=(27+3)+(9-3)
= 30+5	=30+6
= 35	=36

Learning the method of Efficient Calculation

Exercise 4: Problem-posing
exercise(Negative example)

Example	Learner's answer example
27 + 8	27 + 2
=(27+3)+(8-3)	
= 30+5	
= 35	

Make a problem that same calculation cannot be applied

Figure 2. Details of Exercises.

#### 3.3 Diagnosis and Feedback

In each exercise, the system automatically determines whether the answer inputted by the learner is true or false. In addition, the system interactively returns feedback to the learner. If it is wrong, feedback according to the contents of the error.

#### 4. Conclusions

In this research, we designed and developed a problem-posing system for Efficient calculation problems. This system was experimentally used in an elementary school 6<sup>th</sup> graders and the results suggest that the exercise promoted the students to interpret the numerical expressions relationally. The results of the experimental use will be reported by another paper (Enomoto et al.,2018).

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