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Characterization of Different Instantiations of Mathematical Blindness

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Abstract: Twenty-four (24) Grade-11 senior high school students under the STEM track from a private university located in Quezon City took part in this study. Of the 24 participants who took the pre-test, only 23 proceeded with the eye-tracking test, where only 19 data were deemed treatable due to technical issues. This study examined different instantiations of mathematical blindness thru the lens of the Tripartite theory and with the help of the eye-tracking method that collected empirical data and aided in the investigation on how these instantiations manifested in participants' mathematics problem-solving, particularly problems involving quadratic equations, ratio and proportion, geometry, and concept of speed. Other than the answers, solutions, and interviews recorded, quantifiable data were also extracted from the eye-tracker in addition to gaze movement and heat maps. Three instantiations of mathematical blindness were characterized in this study: (1) The Einstellung effect; (2) spurious correlation; and (3) intuitive rule. Among the three, the predominant instantiation of mathematical blindness observed was Spurious correlation.

Keywords: Psychology of Mathematics Education, problem-solving, reasoning, eye-tracking

1. Introduction

Mathematical blindness is defined as a cognitive impairment caused by lack or surplus of attention given to present stimuli (stimuli referring to words, details, or figures that are present in a problem). The differing amounts of attention given to present stimuli may cause the individual to fail, to some degree, in cognition—to process the necessary concepts, and/or procedures to be able to answer or find the efficient solution to a certain problem. The concept of mathematical blindness is not new as some of its aspects have already been studied both in the field of Psychology and Mathematics Education. This study only aims at characterizing how the said phenomenon manifests in students' solution or reasoning about a given problem. In addition, the difference between the concept of inattentional blindness (in psychology) and mathematical blindness is that the former describes only the instance when an individual lacks in giving attention to stimulus or stimuli while the latter describes the event where a person lacks or gives too much attention to given stimuli leading to misuse or overgeneralization of mathematical tools. Tools that are commonly used by students include formulas (e.g. quadratic formula), algorithms (e.g. setting up a proportion), strategies (e.g. identifying key words), and intuitions (e.g. multiplication makes bigger).

Theoretically, mathematical blindness may manifest in mathematics problem solving. It is a phenomenon that may be observed, but not limited to, when an individual i) uses a wrong formula or concept in solving a mathematics problem; ii) fails to use an efficient method of solving on a particular problem; and iii) fails to answer a mathematics problem. Note that lack of mathematical knowledge was not considered as a manifestation of mathematical blindness as the phenomenon is only referring to already known concepts or constructs that an individual failed to perceive.

The researchers made use of the eye-tracking method for collecting data in conducting empirical studies of human perception, cognition, and behavior. As its name suggests, eye-tracking is a means of determining aspects of participant's sensory perception in the visual modality – where they are looking. In this study, the researchers used the Gazepoint GP3 Eye-tracker as a recording instrument for the participants' eye movement which added to the objective analysis of the participants' reasoning, in pursue of characterizing the different instantiations of mathematical blindness, which is the main objective of this study.

Specifically, this study aimed to characterize different instantiations of mathematical blindness based on participants' eye-movements and their solution or reasoning about a given mathematical problem, and also to determine predominant instantiations and classify participants' solutions or reasoning.

2. Theoretical and Conceptual Constructs

This study is anchored on different theoretical and conceptual constructs that elicit similar characteristics of mathematical blindness.

The properties of System 1 and System 2 according to the *Dual-Learning Theory*, and the *Tripartite Theory* (Stanovich, 2011) show features of the two processes that contribute as to why mathematical blindness occurs and was used in assessing the participants' cognition in solving mathematics problems.

The *Einstellung* which is described as the mechanized state of mind (Luchins, 1942) refers to an individual's tendency to solve a given problem in a specific manner even though better methods exist.

Ben-Zeev and Star's (2001) concept of *spurious correlation* is an event hypothesized to occur when a student perceives a correlation between an irrelevant feature in a problem and the algorithm used for solving that problem, and then proceeds to execute the algorithm when detecting the feature in a different problem.

Reminiscent of the above-mentioned concept is the concept of *intuitive rules* (Tirosh & Stavy, 1999). For instance, a student who thought that "the heavier the object the faster it falls" is said to rely on the "More A – More B" intuitive rule.

Lastly, this research utilized the eye-tracking method which has been, over the years, is increasingly being used in research in mathematics education. (e.g., Chesney, McNeil, Brockmole, & Kelley, 2013; Andra, et. Al, 2013; Miroslawa & Rosiek, 2016; Shayan, et. Al, 2017; Schindler, M., Haataja, E., Moreno-Esteva, E. G., Shvarts, A., & Lilienthal, A., 2018). This method was used to quantify visual attention such as time of fixation and saccades (movement of the eye from fixation on one point to another), and measure perceptual range when an individual is solving a mathematics problem.

3. Analysis Results

3.1 Quadratic Equations

In this task, the participants were asked to solve 6 quadratic equations using any method they know, 5 of which are quadratic trinomials and the last, a quadratic binomial. The questions were presented as slides on the screen one after another. The AOIs (Areas of Interest), namely AOI 0, 1, and 2 are designated for each of the terms for the first five equations, and AOIs 0 and 1 for the last equation.

For equations 1, 2, 3, and 5, the equations contain quadratic trinomials that are not in the form of any special products. Equation 4 contains a perfect square trinomial and equation 6 is in the form of a difference of two squares. The aim of this test is to characterize how the participants approached different forms of quadratic equations and identify whether certain instantiations could be observed to cause difficulty in solving the problems.

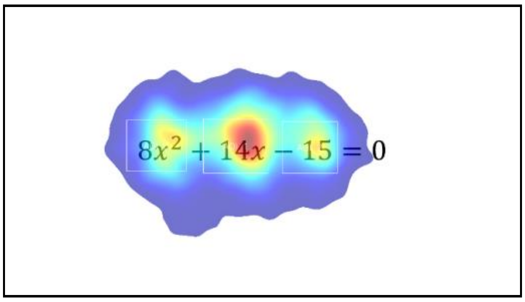

Figure 1. Sample aggregated heat map (equation 1).

Participants' gaze patterns suggest that they give more attention to the middle term when solving quadratic trinomials. The numerical data collected by the eye-tracker also suggests the same as shown below.

Table 1
Recorded data from the eye-tracker for the quadratic equations

AOI Name	Ave Time to 1st View (sec)	Ave Time Viewed (sec)	Ave Fixations (#)	Average Revisits (#)	N
AOI 0 (Leading Coefficient)	4.9642	1.3306	7.2958	6.3758	19
AOI 1 (Middle term)	2.9748	2.6528	13.786	11.1058	19
AOI 2 (Constant term)	6.1278	1.6322	8.5418	7.0908	19

On table 1, AOI 1 has the smallest "average time to 1st view" and the largest "average time viewed", "average fixation", and "average revisit". Further analysis of the gaze patterns, interviews, and written solutions implies that for some of the participants when solving a quadratic trinomial, using factoring as their primary method is more likely. This is evident as the aggregated gaze patterns suggest that the participants used trial-and-error in obtaining the coefficient of the middle term. This means that when students are given a quadratic trinomial, what first comes to mind is to check if the middle term is obtainable thru trial-and-error involved in the factoring method. In this task alone, eight participants used factoring as their primary method in the first four equations.

For this task, the instantiations of mathematical blindness observed are spurious correlation and intuitive rule. First, the spurious correlation for this task was characterized by the participants repeated use of the factoring method or the quadratic formula upon determining whether there is a leading coefficient or not, also upon perceiving whether the coefficients are small or large. Lastly, the intuitive rules that the participants had are the following: (1) since the previous question was solved using the quadratic formula or by factoring, then the succeeding question might also be solved using the same method; and (2) they will use a method that already has become second nature to them when solving quadratic equations.

3.2 Musician Problem

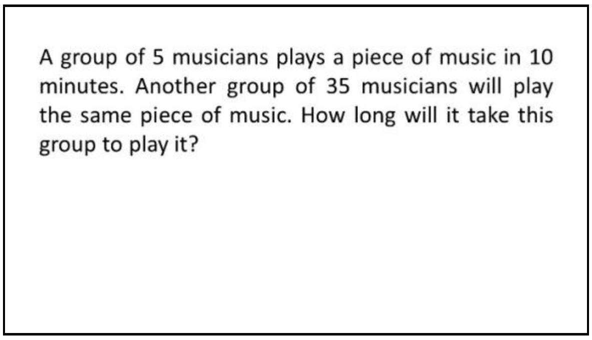

Figure 2. Slide shown to participants for the Musician problem.

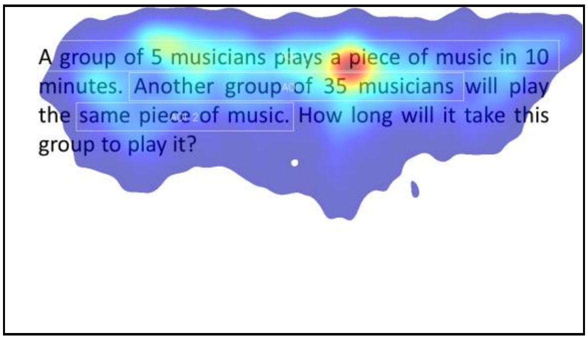

Figure 3. Aggregated heat map for the Musician problem.

AOIs 0, 1, and 2 are designated for the phrases "A group of 5 musicians plays a piece of music in 10", "Another group of 35 musicians" and "same piece of music", respectively.

10 out of 18 participants (1 participant's data was deemed untreatable due to technical issues) used the concept of ratio and proportion in solving the problem where 9 of these participants answered 70, implying that they established a direct proportion between the number of musicians and amount of time to play the music, and 1 participant answered 10/7, who may have established an indirect proportion. These participants who used the same concept twice recognized the feature of the problem:

two units of measurements are identified with a “typical” missing-value proportional question in the end.

Table 2
Data recorded by the eye-tracker for the Musician problem

AOI Name	Ave Time to 1st View (sec)	Ave Time Viewed (sec)	Ave Fixations (#)	Average Revisits (#)	N
AOI 0	1.742	8.006	43.522	29.318	18
AOI 1	1.849	4.441	29.682	24.273	18
AOI 2	3.747	1.832	12	9.762	18

Table 2 shows that the AOI with the least time viewed is AOI 2 which is designated for the phrase “same piece of music”, a plausible cause of why most of the participants did not realize that the problem should not be approached using ratio and proportion. However, some of the participants who gave the correct answer also struggled in realizing that the answer should be 10 minutes. Especially 2 participants, who have shown signs of detachment in reasoning between what is mathematical and realistic. This detachment was elicited when they asked if the problem should be solved “mathematically” or “logically/realistically”.

Again, further analysis of the gaze patterns, interviews, and written solutions tells us that in this case, it is possible that four simultaneous instantiations of mathematical blindness have occurred: the Einstellung effect, spurious correlation, and intuitive rules. First, the Einstellung effect in this problem was characterized by fixation in the two units of measurements that were identified and on the concept of using ratio and proportion as a solution. Second, the spurious correlation was characterized by the overuse of the concept of ratio and proportion upon detecting the two units of measurements. Lastly, the “More A – more B” intuitive rule was evident when the participants established a direct relation between the number of musicians and the amount of time it takes to play the same piece of music.

3.3 Speed Problem

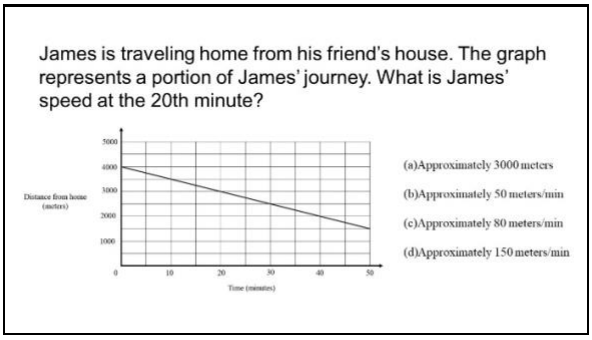

Figure 4. Slide shown to participants for the Speed problem.

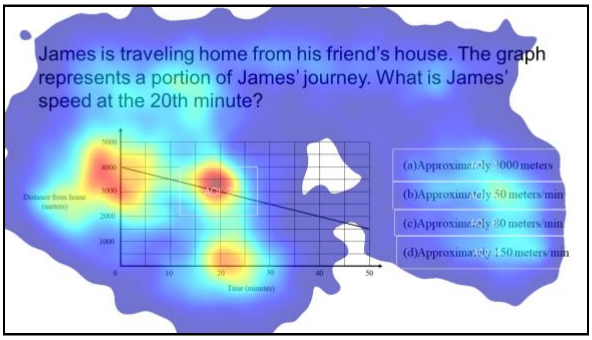

Figure 5. Aggregated heat map for the Speed problem.

Note that in this problem, data from only 18 participants were recorded due to technical issues.

The aggregated heat map shows that the participants’ attention was more directed towards the point in the graph where the 3000 meters distance intersects with the 20th minute time. Quantitatively, the average number of fixations on that point is 12.409 and with average number of revisits 10.905.

Most of the participants interpreted the problem as a speed problem, where 16 participants read the distance from the graph (3000 meters) that corresponds to the 20th minute, applied the s=d/t formula, and obtained 150 by dividing 3000 by 20. Interestingly, only 2 of the 18 participants selected the correct answer “B” which was based on understanding speed as the ratio of change in distance to change in time.

Since the formula s=d/t is a commonly used formula in solving problems involving speed, it has become a tool that can be inappropriately used in solving mathematical problems most especially when their understanding of the problem or the use of the formula itself is superficial. In this task, the participants who gave “D” as an answer correlated the use of the formula to the given values or to what was being asked in the problem.

Table 3
Recorded data by the eye-tracker for the Speed problem

AOI Name	Ave Time to 1st View (sec)	Ave Time Viewed (sec)	Ave Fixations (#)	Average Revisits (#)	N
AOI 2	16.102	2.21	12.409	10.905	18
AOI 3	16.672	1.146	7.429	5.8	18
AOI 4	36.186	1.374	6.4	4.833	18
AOI 5	18.555	1.35	7.13	5.818	18
AOI 6	29.954	1.023	7.05	6.053	18

Recall that most of the participants gave “D” as an answer, hence, the AOI designated for choice “D” (AOI 4) should have gained the highest attention. However, the data above show otherwise.

The instantiations of mathematical blindness observed here is spurious correlation. The spurious correlation was characterized by the participants’ overuse of the concept of speed, specifically the use of the formula s=d/t upon detecting that there are two corresponding values of distance and time. Their use of this concept was also deemed superficial because they used the concept, however, in a wrong understanding of the question.

4. Conclusion

Based on the results gathered, this study concludes that mathematical blindness can manifest in students’ mathematics problem-solving and reasoning in different ways. These manifestations can be caused by objects (stimuli) in a problem or by misconceptions. These were observed especially when a participant overuses concepts upon detecting a certain feature in the problem. It is also conclusive that through the use of the eye-tracker, the analysis of their reasoning became more objective with the help of the quantifiable data and visualizations provided. This study led to the realization that in solving mathematical problems, it is not just reasoning alone that influences how a problem is approached, but also by objects that can be perceived in the problem itself.

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