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Learning by Problem-Posing as Kit-Building for Structure Understanding of Polynomial Factorization

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Abstract: Polynomial factorization is a basic skill required of middle-school students. Students typically learn the skill through solving polynomial factorization problems. In this paper, as a new method to learn the skill, we propose learning by problem-posing factorable polynomial expressions. In this problem-posing, we use the kit-building method, where a student is provided with a set of components such that a structure can be constructed using the components. To learn polynomial factorization, it is crucial to understand the polynomial expression as a structure. The effectiveness of structure understanding in the kit-building method is confirmed in several learning domains, for example, arithmetic and mathematic word problems. In the exercise performed in this study, a student is required to create a polynomial expression that can be factorized using a specified factorization rule, which we call "problem-posing." Students and teachers accepted that this is as a better form of learning factorization, i.e., through practical use at universities and middle schools.

Keywords: problem-posing, polynomial factorization, learning environment, kit-building method

1. Introduction

In this paper, we present a learning environment in which a learner learns factoring polynomial by problem-posing. Two kind of case studies are described: "the practical use for university students" and "the case study for middle school students". The focus of this study is the polynomial factorization in middle schools. The factorization used in this study involves factoring a polynomial with factors such as "$x^2 + 5x + 6 = (x + 2) (x + 3)$." Therefore, learning factorization means understanding the conditions for factoring such a polynomial or expanding the factorized expression into a polynomial. In general, polynomial factorization is learned using problem-solving exercises. To perform effective problem-solving exercises, several learning environments have been proposed (Kim & Glass, 2004; AbuEloun & Abu-Naser, 2017).

Problem-posing has been suggested as an effective learning method for understanding the solution instead of problem-solving (Polya, 1945; Silver & Cai, 1996). This practice has been adopted in various learning environments, and its variations have been developed as well (Yu et al., 2018; Chang et al., 2012; Nakano et al., 1999). We are continuously investigating a learning environment for problem-posing based on problem structures (Hirashima & Hayashi, 2016). In our learning environments, the structure of the problem is defined, and the decomposed one is assigned to the learner as a kit. Learners can deepen their understanding of the problem structure by building the kits (components) through trial and error while receiving feedback from the learning environment. We call this learning method the kit-build method. In this study, we applied a kit-building method for problem-posing of polynomial factorization and developed a learning environment. This described in Section 2. We also reported the results of two case studies in Section 3 and 4. Section 5 is conclusion.

2. Suggested Problem-posing of Polynomial Factorization

2.1 Difficulty of Problem-posing of Polynomial Factorization

Regarding polynomial factorization, problem-posing is a promising learning activity for the same reason, but it is not easy to realize it. The reason is that in the factorization if a student first considers the factorized answer, the student is able to expand it to create a problem. The expansion is usually much easier than the factorization. Practical research of problem-posing of polynomial factorization in a classroom reported that students were able to make several complex problems of polynomial factorization which seemed too difficult for the students to solve (Okiyama, 2011). The result suggests that the students often posed problems by expanding answers. The research concluded that it was not easy to conduct problem-posing of polynomial factorization as a learning activity.

We have continued to develop an original learning environment by problem-posing of the kit-build method. The effectiveness of structure understanding using the kit-building method has been confirmed in several learning domains, e.g., arithmetic and mathematic word problems (Yamamoto & Hirashima, 2017). Therefore, we propose the factorization question learning based on the kit-building method as an effective learning method.

2.2 Suggested Problem-posing of Polynomial Factorization and its Learning Environment

Table 1 shows each step of the suggested exercise. In Step 1 of this exercise, the learner performs the factorization using the formula. This step is intended to review the formula. Figure 1 (a) is shown an interface of exercise for Step 1. The learner inputs and deletes an answer by tapping the blank box to the right of “Equal” and pressing each input button. If the learner taps the diagnosis button in Steps 1, two type of feedbacks are returned: “correct answer” and “calculation error.”

In Step 2, the learner must change the assigned polynomial to a polynomial to which the proposed factorization formula can be applied. The learner is allowed to change the coefficients of the polynomial. A set of changeable coefficients is provided to the learners. As this polynomial is a factorization problem, we call this exercise “problem-posing.” In this activity, the learner must be aware of the conditions under which the assigned factorization formula is applicable. In Step 2, if the learner changes one coefficient appropriately, he/she will obtain the correct answer. In this step, the learner is required to factor the changed polynomial in addition to changing the polynomial. In the step 3 exercise, the learner needs to change some coefficients.

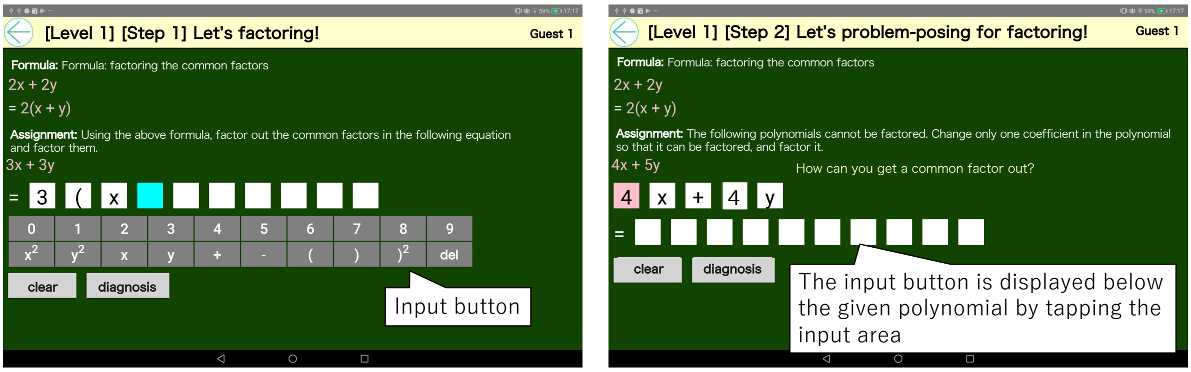
Figure 1 (b) is shown an interface of exercise for Step 2, 3. In this interface, an input area for changing the assigned polynomial is added below the assigned polynomial. When the learner taps the coefficient in this area, a number that can be selected is presented in a pull-down format. By selecting a specific number, the learner can change the assigned polynomial. After changing the assigned polynomial such that it can be factored, the learner inputs the factorized formula similarly as in Step 1. If the learner touches the diagnosis button in Steps 2 and 3, three feedbacks are returned: “correct answer,” “calculation error,” and “polynomial change error.”

This exercise lets learners change non-factorable problems to factorable problems and factor the changed problem for verifying. Through these change activities, the student is promoted to consider the conditions of application of the solution based on the structure of polynomial expressions. To let a student focus on the differences between the original problem and a posed problem, the student is provided components for the change and requested to pose a new problem by using the components. Therefore, they cannot pose a proper problem without explicitly being aware of the factors through the operation of the component. By performing this exercise in each formula, the learner will likely

Table 1. *Three Steps of Our Learning Method*

	Given Question	Example	Answer
Step 1	Polynomial Factorization	3x+3y = ?	3(x+y)
Step 2	Change Polynomial (one coefficient) & Polynomial Factorization	4x+5y = ? (select 4 or 5)	4x+**4**y = 4(x+y)
Step 3	Change Polynomial (some coefficient) & Polynomial Factorization	4x+5y = ? (select from 2 to 9)	**7**x+**7**y =7(x+y)

be cognizant of the conditions for applying each factorization formula. This study targets middle school students. Therefore, five formulas used in the learning environment are as follows: $ax + ay = a(x+y)$; $x^2 + (a + b) x + a*b = (x + a) (x + b)$; $x^2 - a^2 = (x + a) (x - a)$; $x^2 + 2*a*x + a^2 = (x + a)^2$; and $x^2 - 2*a*x + a^2 = (x - a)^2$. These five formulas were implemented in the learning environment as Levels 1-5. The students learn at each level through the exercises in Steps 1-3.



(a) Interface of Exercise in Step1 (b) Interface of Exercise in Step2 and 3

Figure 1. Interface of exercise in learning environment.

3. Experimental Use at University

3.1 Procedure

The participants were 17 undergraduate and graduate students in the engineering department of the university. We would like to verify that the suggested exercise is useful for learners who have already learned factorization. The experiment was a pre-post method, and tests were performed before and after using the learning environment. Participants answered the questionnaire after the test. The times for each activity are as follows: The pretest and post-test required 19 min; the use of the learning environment required 20 min; and providing answers to the questionnaire required 5 min.

In this exercise, participants were required to answer the tasks of levels 1 and 2 only owing to time limitations. If participants had time remaining to use the learning environment, we instructed them to learn repeatedly through assignments from levels 1 and 2.

3.2 Pre-test and Post-test

We conducted three pre-tests and post-tests. The first one is a factorization problem solving test. The easiest problem is of the same difficulty as the problem implemented in the learning environment. Other problems are more difficult than those implemented in the learning environment. These kinds of problem are used in high school learning. The second test is the problem-posing of the factoring polynomial test. This test is the same type as the exercise performed by the learner in the learning environment. The participant was assigned two polynomials. The learner was required to change each assigned polynomial to pose four factorizable problems in each. The participant was allowed to change only one coefficient when posing a polynomial that can be factorized. The third one is a factorization problem-posing test from scratch. The participants were required to pose four problems.

3.3 Results

First, we present the log analysis of the learning environment. The system log was obtained from 17 participants. From the analysis result, it was discovered that all 17 participants completed the level 1 and 2 exercises. The average number of correct answers and error were 8.65 and 3.94.

Second, the factorization problem-solving test did not differ significantly different between the pre- and post-tests (Wilcoxon signed-rank test, $p = .343 > .05$). The percentage of correct answers increased only slightly from 48% to 54% overall. However, the correct answer rate was 90% for the simplest problems that were implemented in the learning environment. The percentage of correct answers increased only slightly from 47.5% to 53.8% in high difficulty problems. This result showed that the participants successfully solved the difficult factorization.

Third, we show the results of problem-posing in the factoring polynomial test and the factorization problem-posing test (Table 2). Here, we investigated the number of posed problems that can be solved by a formula other than the one used in the exercise (called advanced problem). The number of this advanced posed problems is significantly different between the pre- and post-tests (Wilcoxon signed-rank test, $p = .004 < .05$). In the factorization problem-posing test, a significant difference was observed in the number of problems posed between the pre- and post-tests (Wilcoxon signed-rank test, $p =.011 < .05$). Furthermore, the number of posed advanced problem between the pre- and post-tests differed significantly (Wilcoxon signed-rank test, $p =.005<.05$). These results showed that the participants successfully posed the difficult factorization problem.

Finally, the questionnaire and its results are shown in Figure 2. This questionnaire was implemented based on a four-point scale (strongly agree, almost agree, almost disagree, and strongly disagree). While the feedback was not agreed highly, the usefulness of the learning environment for factorization learning was highly evaluated.

From the results of this practical use, we concluded the following:(a) the suggested exercise was useful for learning factorization for university students who have already learned factorization; (b) the suggested exercise may promote the learner to awareness for transfer. This result suggests that learners can deepen their understanding of the factorization formula by our leaning environment.

Table 2. *The Score of Each Problem-posing Test*

Test type	Value type	Pre-test	Post-test	p-value	
Problem-posing in the factoring polynomial test	Number of posed problems	3.88	4.35	0.14	n.s.
	Number of posed advanced problems	1.53	2.00	0.04	*
Factorization problem-posing test	Number of posed problems	1.88	2.59	0.011	*
	Number of posed advanced problems	0.71	1.12	0.049	*

*$p<.05$

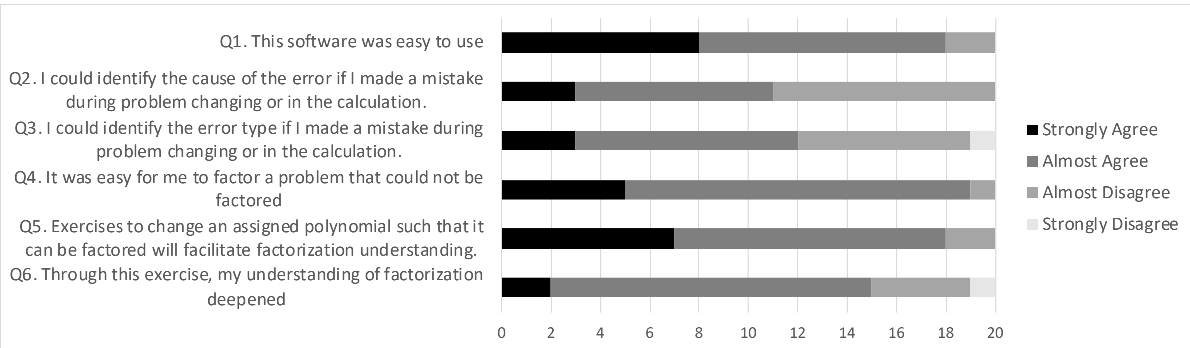


Figure 2. The contents and answers of the questionnaire.

4. Case Study at Middle School

4.1 Procedure

After conducting the experimental use, we explained our learning environment to middle school teachers. We decided to conduct a case study based on 37 third-grade middle-school students, who were the better problem solvers among all third graders in a particular school. Furthermore, they have already learned polynomial factorization. The purpose of this case study is to verify that the target learner can learn factorization in the proposed learning environment.

In this case study, we first explained the method to use the system and provided an Android tablet within approximately 10 min. Next, the students learned polynomial factorization in our learning environment for approximately 30 min; finally, they answered the questionnaire in 5 min. This exercise comprised only 15 problems with five levels. The learners who completed the answer were asked to use this system again or to use another system. As using another system does not affect the analysis of this case study, we will omit the explanation of the other system used. In addition, when students decided to use our learning environment again, they were instructed to consider another answer.

4.2 Time of exercise

We were able to collect the appropriate logs for 36 students. The average time taken for this exercise was 23 min 33 s, including the time for a second learning. Figure 3 shows the distribution of the learner's practice time in our learning environment. The usage time was polarized significantly, and we speculated that the quality of the exercises differed by group. Therefore, 13 students who had been learning beyond the average usage time were grouped as the long-time usage group (L-group), whereas 23 students who required less than the average usage time were grouped as the short-time usage group (S-group). In addition, by investigating the number of diagnoses performed every 10 min in each group, we discovered that approximately 10 - 14 times of diagnosis was performed in each term. This fact suggested that the student has learned continuously in this case study.

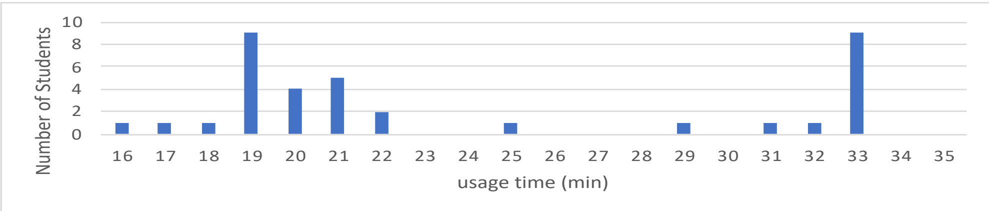


Figure 3. Learning Environment Usage Time and Number of Participants in Each Time.

4.3 Result of Number of Correctness and Incorrectness on Learning Environment

The percentage of correct answers in all exercises using our learning environment were 72.2% and 87.3% in the L-group and S-group, respectively. A significant difference was observed between these percentage of correct answers rates (Wilcoxon signed-rank test, $p = .0002 < .01$). In addition, the percentage of correct answers in the first and second trials of each group was analyzed, in which those of the L-group were 71.1% and 79%, respectively. Meanwhile, those of the S-group were 88.5% and 87.0%, respectively. No significant difference was observed between the first and second correct accuracy rates in each group. The correct answer rates in first trial are shown in the order of Step 1, Step 2, and Step 3 of each level, where those of the L-group were 93.1%, 80.1%, and 83.8%, respectively; and 98.3%, 92.2%, and 91.6%, respectively, for the S-group.

Finally, we describe the number of errors generated by the learner in our learning environment. A total of 143 errors were discovered in all the students' exercises. The details are shown in Table 3. Three types of errors occurred: "error of method for changing polynomial," "error of factoring polynomial," and "error of changing polynomial." The first one is the error where the polynomial is not changed as shown in the assignment. For example, the assignment requires changing two coefficients, but the learner does not change any of them. The second one is an error in polynomial factorization. The third one is an error where the polynomial change is wrong. This error includes cases where the learner has posed a problem that cannot be factored or that cannot be solved with the given formula.

Table 3. *Total Number of Each Errors in Each Group*

	Error of method for changing polynomial			Error of factoring polynomial			Error of changing polynomial		
	Step 1	Step 2	Step 3	Step 1	Step 2	Step 3	Step 1	Step 2	Step 3
Long-term use	-	31	11	11	10	1	-	14	16
Short-term use	-	10	14	4	3	3	-	10	5

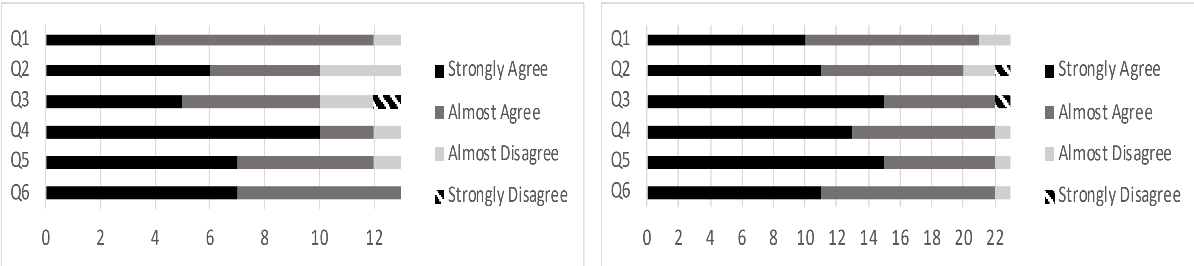
4.4 Questionnaire Results

Finally, the questionnaire results are shown in Figure 4. The contents of questionnaire are the same as that shown in Figure 2. For almost all questions, the S-group and L-group answered positively.

Immediately after the class, we sought the opinions of two teachers, i.e., a mathematics teacher and a class teacher, regarding the practical use of the exercises. They reported that the exercises were meaningful for understanding factorization, and that such exercises would be impossible without our learning environment. Moreover, the teachers stated that assigning one lecture period to solve the exercises of this system would be worthwhile.

Compared with the performance of the S- group, that of the L-group was lower. The L-group had more errors than the S-group, and the exercise time was longer. However, the S-group more appreciates our learning environment than the L-group. This result suggests that our learning environment may be more useful in a slow learner.

From these results, it was confirmed that both the learners and teachers recognized that the exercises of the proposed learning environment were useful for understanding factorization.



(a) Answers for Long-term Use Group (b) Answers for Short-term Use Group

Figure 4. Results of questionnaire.

5. Conclusions and future works

In this paper, we proposed a learning environment for problem-posing a polynomial factorization by kit-build method. Furthermore, we reported the results of practical use for university students and of case studies for middle school students. The results of these case studies have been suggested that our learning environment may be effective in understanding mechanism of factorization.

For future studies, we are considering improving the feedback of learning environment. Furthermore, we plan to conduct a case study based on an experimental group and a control group and a case study targeted a slow learner for verifying learning effect of our learning environment.

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