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Computational Thinking Activities in Number Patterns: A Study in a Singapore Secondary School

Shiau-Wei CHAN[a*], Chee-Kit LOOI[a], Weng Kin HO[a], Wendy HUANG[a], Peter SEOW[a], Longkai WU[a] & Mi Song KIM[b]
[a]*National Institute of Education, Nanyang Technological University, Singapore*
[b]*University of Western Ontario, Canada*
*shiauwei.chan@nie.edu.sg

Abstract: In recent decades, there is a growing body of literature about CT and mathematics education. Thus, the purpose of this study was to identify the effectiveness of the integration of CT into mathematics classrooms in a Singapore secondary school, particularly in the topic of number patterns. A quasi-experimental design was employed in this study. 106 lower secondary students were involved in this study. 70 of them were assigned into the experimental group, while 36 of them were assigned into the control group. The students in the experimental group were given intervention unplugged Math+C activities and plugged Math+C activities using a spreadsheet. Meanwhile, the students in the control group received no intervention and were involved in traditional instruction. Both groups were given a pretest before the instruction and posttest after the instruction. The data obtained were analyzed using a two-way mixed-design ANOVA analysis. The findings revealed that there was a significant main effect of the pretest and posttest between the students from the experimental group and control group, but not a significant main effect of groups. Also, there was not a significant interaction between tests and groups. This study contributes to the area of integration of CT and mathematics in the instruction.

Keywords: Computational thinking, number patterns, secondary school, Math+C, quasi-experiment

1. Introduction

CT has been recognized as a collection of skills and understandings essential for new generations to be capable of using tools, but also able to understand the implication of their competences and limitations (Magana & Silva Coutinho, 2016). In 2014, Singapore launched the Smart Nation initiative, which has among its goals, developing its citizens' digital literacy through coding or CT education for pre-schoolers to adults. In primary school, 10-hour coding classes have been made compulsory in the primary grades. Computing subjects are offered as part of the formal curriculum in secondary schools and junior colleges. The Infocomm Media Development Authority (IMDA) has launched several informal programs to develop CT skills and coding abilities including the Code for Fun Enrichment Programme, PlayMaker program, and Digital Maker program. The Science Centre also provides similar programmes to the general public.

Nowadays, there has been a rising interest in introducing CT into the mathematics classroom. Ho et al. (2017) reported on lessons designed to enrich science and mathematics through the integration of CT. In this study, Math+C is defined as the integration of Math and CT in the design and enactment of lessons. Thus far, the term CT has no universally accepted definition as a field by itself or as a field in the different disciplines like mathematics. In our present context, we define CT practices in the mathematics as involving decomposition (is the process by which the mathematics problem is broken down into smaller sub-problems or sub-tasks), pattern recognition (the action of looking out for common patterns, trends, characteristics or regularities in data), abstraction (the process of formulating the general principles that generate these recognized patterns), and algorithm design (the development of a precise step-by-step recipe or instructions for solving the problem at hand as well as a problem similar to it).

Various studies using experimental or quasi-experimental design methodologies have suggested positive correlations between CT and mathematics learning performance. Calao et al.'s (2015) study found that by using a Scratch visual programming environment to develop computational thinking, students in the experimental group can improve their performance in the mathematical process of reasoning, modeling, reasoning, exercising, and problem-solving. The study of Fidelis Costa, Sampaio Campos, & Serey Guerrero (2017) claimed that the development of math questions that are more aligned with CT can have a positive impact on students' problem-solving ability. These studies provide more evidence for the development of CT in mathematics education as a mechanism to promote students' capabilities. Hence, it is hypothesized that the quasi-experimental integration of CT and mathematics did in this study would have a positive impact on the learning performance of secondary students in Singapore. Thus, this study aims to investigate the impact of Math+C activities on the performance of secondary students in a Singapore school. There were two research questions being examined in this study: (1) Is there a statistically significant main effect of the pretest and posttest between the experimental and control group? (2) Is there a statistically significant interaction between tests (pretest and posttest) and groups (experimental group and control group)?

2. Method

2.1 Participants

This study employed a quasi-experimental design that non-randomly assigned students into the experimental group and control group. A total of 106 Secondary One students from a Singapore secondary government school involved in this study where 70 of them in the experimental group and 36 of them in the control group. There were 37 males and 33 females in the experimental group, as well as 20 males and 16 females in the control group. They came from three intact classes. The students already had the prior knowledge on how to evaluate an algebraic expression given an expression and an unknown, as well as on how to solve linear equations with one variable. The experimental group took part in the intervention with unplugged Math+C activities and plugged Math+C activities using a spreadsheet. Meanwhile, the control group was given traditional instruction. Nonetheless, both groups were administered with the pretest before the instruction and posttest after the instruction.

2.2 Instrument

The pretest and posttest were constructed by the researchers. Each test consisted of eight items as revealed in Table 1. The topic covered in both tests was Number Patterns, which was Chapter 7 in the Singapore Secondary Mathematics curriculum. There were three types of number sequences used in this study, i.e. arithmetic sequence, quadratic sequence, and geometric sequence. There were two categories of number sequences presented to students, namely numeral and figural. The validity and reliability of the instruments were determined by using the Rasch model. The construct validity for both tests was good as the values of raw variance explained by measures were greater than 40% (Linacre, 2012). Meanwhile, the reliability for both tests was high, i.e. more than 0.90 (Qiao, Abu, & Kamal, 2013).

Table 1. *Test Items*

Type	Skill Tested	Sample Items
Arithmetic sequence (Numeral)	Identify terms of simple number sequences when given the initial terms	Fill in the missing terms in the following sequence: 7, ____, -5, -11, ____, -23
Quadratic sequence (Numeral)	Identify terms of quadratic sequences when given the rule	Find the 9^{th} term of the sequence: 1, 3, 6, 10, …
Geometric sequence (Numeral)	Identify terms of geometric sequences when given the rule	Write down the next two terms of the number sequence: 9, 27, 81, 243, ______, _______

<table>
<tr><td>Arithmetic sequence (Figural task with successive configurations)</td><td>(a) Use visual cues established directly from the structure of configurations to illustrate the pattern.
(b) Identify terms of arithmetic sequences when given the rule
(c) Generate the rule of a pattern
(d) Obtain an unknown input value when given the formula and an output value</td><td>The diagram below shows a sequence of bricks.

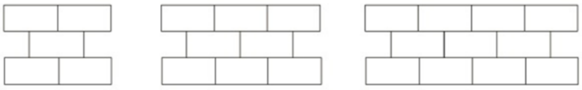



Figure 1 Figure 2 Figure 3

(a) Draw Figure 4.
(b) Complete the following table.
<table><tr><th>Figure Number</th><th>Number of bricks</th></tr><tr><td>1</td><td>5</td></tr><tr><td>2</td><td>8</td></tr><tr><td>3</td><td>11</td></tr><tr><td>4</td><td></td></tr><tr><td>5</td><td></td></tr><tr><td>6</td><td></td></tr></table>(c) Is there a Figure Number in the sequence that contains 136 bricks? Justify your answer.</td></tr>
<tr><td>Arithmetic sequence (Numeral)</td><td>(a) Use only cues established from any pattern when listed as a sequence of numbers or tabulated in a table.
(b) Generate the rule of a pattern</td><td>Consider the following number pattern:
$1 = 3 \times 1 - 2 \times 1$
$8 = 3 \times 2^2 - 2 \times 2$
$21 = 3 \times 3^2 - 2 \times 3$
$40 = 3 \times 4^2 - 2 \times 4$
…
$341 = 3 \times n^2 - 2 \times n$

Write down the equation in the 6^{th} line of the pattern.
Deduce the value of *n*. Explain or show how you figured out.</td></tr>
</table>

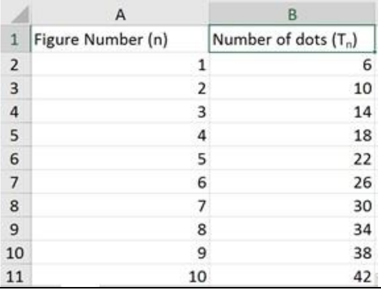
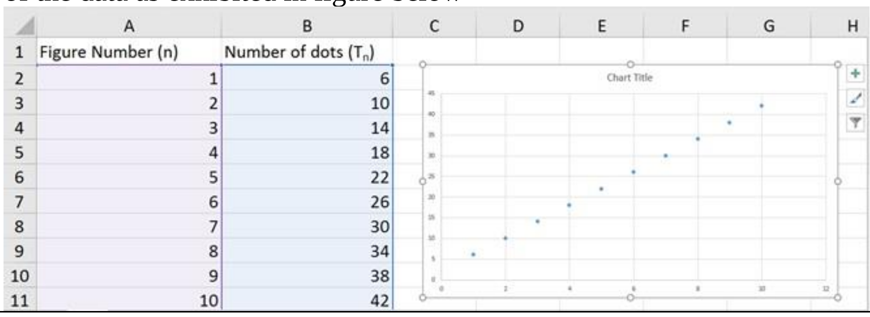
2.3 Computational Thinking Activities for Experimental Group

For the experimental group, the students were given intervention with unplugged Math+C activities and plugged Math+C activities using a spreadsheet. Four CT practices, i.e. pattern recognition, decomposition, algorithm design, and abstraction were employed throughout the instruction. The Math+C worksheets used in the unplugged Math+C activities were designed in line with these four CT practices.

Table 2 displays the instructional design of the plugged Math+C activities using the spreadsheet. There are four design principles for the integration of CT into lessons of number pattern based on CT practices, namely the principle of complexity, the principle of data, the principle of mathematics, and the principle of computability (Ho et al., 2020). The first principle is the principle of complexity. The tasks related to the topic of learning digital patterns are very complex to be broken down into subtasks. If the task is routine and can be easily solved using simple and well-known methods, decomposition cannot be utilized well.

The second principle is the principle of data. This task should consist of quantifiable and observable data that can be utilized, transformed, processed, and stored. Furthermore, the principle of mathematics is regarded as the third principle. We have to determine whether the task causes a situation or problem that can be mathematized. Mathematics is a problem constructed using mathematical terms. It includes altering the actual problem context to a mathematical problem in a precise and abstract technique. The task ought to be formulated abstractly so that it can be reasonably reasoned, described, and represented. The last principle is the principle of computability. We need to ensure a task solution that can be performed on a computer through a finite process.

Table 2. *Sample of instructional design for plugged Math+C activities using a spreadsheet*

<table>
<tr><th>CT Practices</th><th>Sample of Instructional Design</th></tr>
<tr><td>Algorithm design</td><td>The students utilized the spreadsheet to perform recursion method and attained the final product as shown in figure below

<table>
<tr><th></th><th>A</th><th>B</th></tr>
<tr><td>1</td><td>Figure Number (n)</td><td>Number of dots (T_n)</td></tr>
<tr><td>2</td><td>1</td><td>6</td></tr>
<tr><td>3</td><td>2</td><td>10</td></tr>
<tr><td>4</td><td>3</td><td>14</td></tr>
<tr><td>5</td><td>4</td><td>18</td></tr>
<tr><td>6</td><td>5</td><td>22</td></tr>
<tr><td>7</td><td>6</td><td>26</td></tr>
<tr><td>8</td><td>7</td><td>30</td></tr>
<tr><td>9</td><td>8</td><td>34</td></tr>
<tr><td>10</td><td>9</td><td>38</td></tr>
<tr><td>11</td><td>10</td><td>42</td></tr>
</table></td></tr>
<tr><td>Pattern recognition</td><td>The students drew a scatter plot to represent the data to reveal the patterns of the data as exhibited in figure below

<table>
<tr><th></th><th>A</th><th>B</th></tr>
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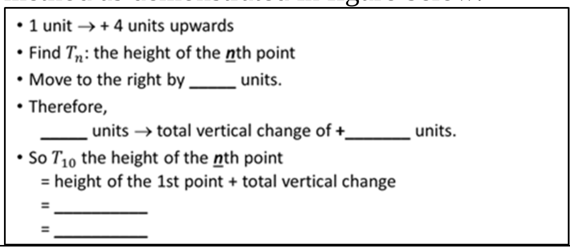
<tr><td>6</td><td>5</td><td>22</td></tr>
<tr><td>7</td><td>6</td><td>26</td></tr>
<tr><td>8</td><td>7</td><td>30</td></tr>
<tr><td>9</td><td>8</td><td>34</td></tr>
<tr><td>10</td><td>9</td><td>38</td></tr>
<tr><td>11</td><td>10</td><td>42</td></tr>
</table>
</td></tr>
<tr><td>Decomposition</td><td>The students were guided to do the decomposition by breaking the problem into two smaller problems:
Problem 1: What is the starting number? Where to key in?
Problem 2: How do I use the recursion method to generate the number pattern?</td></tr>
<tr><td>Abstraction</td><td>The students were asked to generate the general formula using abstraction method as demonstrated in figure below.

</td></tr>
</table>

2.4 Lessons for Control Group

The control group was instructed using a more traditional approach. Unlike the experimental group, the students in the control group did not utilize any technology to learn the topic of number patterns. Furthermore, the worksheets used for the control group were routine problems, which was different from the worksheets that employed in the experimental group. The teacher used two worksheets when she taught the students in the control group. The students were asked to find the next two terms in the number sequence. Besides, the students were given a general term for a sequence of numbers, and they were required to find the term for the sequence. They were guided on how to generate general terms from a sequence of numbers in a worksheet.

3 Findings

Raw data are not always linear, so they should be converted to linear measures before conducting the parametric statistical test such as t-tests and ANOVA. In this study, the researcher utilized the Rasch model analysis to convert the raw data into linear measures, which means the scores were transformed into an interval scale. The person measures obtained in the Rasch analysis were then employed in the two-way mixed-design ANOVA analysis. This was to avoid problems related to the nonlinearity of raw test data and rating scales (Boone, Staver, & Yale, 2014).

The mean scores of pretest for the experimental group and control group were identified to ensure the students from both groups had equivalent capabilities in answering the questions to get rid of threats and support research (Gay, Mills, & Airasian, 2011). As shown in Table 3, the experimental group had a mean pretest score of 1.2183 (SD = 0.7250), while the control group had a mean pretest score of 1.2172 (SD = 0.5316). This indicated that the capabilities of the students from both groups were identical. The mean posttest scores for the experimental group were 1.4941 (SD = 0.7819) and the mean posttest scores for the control group were 1.4769 (SD = 0.7221).

Table 3. *Descriptive statistics of pretest and posttest*

Test	Experimental Group (N = 70)		Control Group (N = 36)	
	M	SD	M	SD
Pretest	1.2183	0.7250	1.2172	0.5316
Posttest	1.4941	0.7819	1.4769	0.7221

A two-way mixed-design ANOVA was conducted to address research questions 1 and 2. From the tests of within-subjects effects in Table 4, it was noticed that there was a significant main effect of the pretest and posttest between the students from the experimental group and control group with F (1, 104) = 8.511, p = 0.004 < 0.005. Nevertheless, there was not a significant main effect of groups by referring to tests of between-subjects effects in Table 4. This is because F (1, 104) = 0.006, p = 0.937 > 0.05. Furthermore, there was not a significant interaction between tests (pretest and posttest) and groups (experimental group and control group) with F (1, 104) = 0.008, p = 0.930 > 0.05 as revealed in table 4.

Table 4. *The analysis results of two-way mixed design ANOVA*

Source	SS	df	MS	F	P
Group	0.004	1	.620	.006	.937
Test	3.410	1.000	3.410	8.511	.004
Group*Test	.003	1	.003	.008	.930
Errors					
SS_{BS}	64.921	104	.624		
SS_{WS}	41.667	104	.401		
Total		106			

Note. ****p* < .001

4 Discussions and Conclusion

For research question one, the results showed that there was a significant main effect of the pretest and posttest between the students from the experimental group and control group based on tests of within-subjects effects. This indicated that the pre-test and post-test of the experimental group and the control group are significantly different. However, there was not a significant main effect of groups by referring to tests of between-subjects effects. This implied that both groups did not make significant improvements in the posttest. For research question two, there was not a significant interaction between tests (pretest and posttest) and groups (experimental group and control group). It means that the changes from pretest to posttest for the experimental group and control group were somewhat similar. In other words, the intervention with unplugged Math+C activities and plugged Math+C activities using a spreadsheet had not much impact on the students' performance in mathematics.

The findings did not support the hypothesis that integrating CT in lessons can result in improved mathematics learning. It means that the results of this study were not consistent with the results of previous studies with a positive impact of CT on students' understanding of mathematical knowledge such as Calao et al. (2015) and Fidelis Costa, Sampaio Campos, and Serey Guerrero (2017). The possible reason why the involvement of students in the CT activities was not effective might due to the short duration of the CT activities given to the students in the experiment group. There was only one session of plugged Math+C activities using a spreadsheet with a duration of less than one and a half hours and unplugged Math+C activities with a duration of fewer than two hours. The experimental students may only engage in certain aspects of the problem-solving process during the CT activities, but these aspects are not sufficient to reveal significant differences with the control students. A follow-up interview or future study ought to be conducted to find out if there is a warrant to suggest that CT elements of the Math+C treatment influenced student learning.

The results obtained added the empirical evidence to the literature review on the incorporation of CT in mathematics and served as a guideline for the researchers, instructors, and school administrators in planning and designing the Math+C lessons in the schools. However, there were some limitations in this study. The sample involved in this study may not be representative of the population of lower secondary students in Singapore. Hence, a larger sample ought to be included in further studies to have a better generalization of the results. Another limitation was the short period of the intervention. So, the duration of the intervention should be expanded in future studies to evaluate the long-term impact on the students' learning gains in the CT concepts and mathematical knowledge. In future studies, it is suggested to expand this study with the respondents from different demographics such as grade level, gender, race, school, computational tools, and so forth.

Acknowledgements

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